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# National Litigation Consultants' Review

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# Applying Benford's Law in Financial Forensic Investigations

By Mark S. Warshavsky, CPA/ABV/CFF, MBA, CVA, CBA, CFE, DABFA, CFFA

Fortunately for the forensic accountant, people that try to conceal fraud tend to develop nonconforming digit (number) patterns that will not generate random numbers thus violating the pattern of randomness.

### Use of Digital Analysis

Using digital analysis will provide a way to detect those patterns. Digital analysis is an innovative method of analyzing groupings of data in an effort to identify numbers within a data set that have nonconforming patterns and deviations. Digital analysis techniques afford the forensic accountant many potential computer-related applications to effectively perform financial forensic investigations. A significant positive result of utilizing digital analysis is the ability to take an unbiased look at 100% of the data set, which should help the forensic accountant to better plan the scope of the engagement. The analysis of digital frequencies may also be appropriate in situations where a forensic accountant needs a quick initial measurement as to whether tabulated data appears to be natural and not manipulated.

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*Business Valuation for the Litigation Practitioner*

# One Path to Reasonable Certainty

by Ricardo Zayas, CPA, CVA, CFE

In litigation, we are expected to opine within a "reasonable degree of professional certainty." In valuation, that "certainty" is developed by competently applying appropriate steps in valuation approaches.

Recently, I observed the testimony of an opposing expert at arbitration proceedings arising from the dissolution of a professional business arrangement. At issue was the value of a 50% stock interest in a professional practice. As expected, the expert discussed his valuation approach, the standard of value he applied, his analysis of the subject business financial data, and set forth his conclusions. As I listened to his testimony, I began to ponder the interplay between certain aspects of his analysis and his ultimate conclusion, which he expressed to a "reasonable degree of professional certainty."

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**MANAGING EDITOR**

P. Dermot O'Neill, CPA/ABV/CFFA, CVA
*EisnerAmper, LLP*
*Philadelphia, PA*



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## History of Benford's Law

One of the most popular digital analysis procedures is an analytical technique known as Benford's Law. It is based on a theory that there are expected frequencies, or occurrences, of digits in a list or data set that was randomly generated, e.g. a general ledger. This technique, which was first identified in the late 1880s by an astronomer and mathematician named Simon Newcomb, was developed by Frank Benford in the 1920s when he was a physicist for the General Electric Company. Benford's Law provides a unique method of data analysis, allowing the forensic accountant to identify fraud, manipulative bias, processing inefficiency, errors, and other non-conforming abnormal patterns as it applies to a company's accounting records.

Benford's discovery began when he noted that the first few pages of logarithm table books were more worn than the later pages. As an example, the logarithm table pages beginning with the digits 1 and 2 were more worn than the table pages with the digits 5 and 6. In those pre-calculator days, logarithm tables in books were used as a method to multiply two large numbers. It was concluded by Benford that he and his fellow employees required the use of pages with numbers of lower first digits more frequently and, therefore, more numbers with lower first digits existed. He reasoned this theory after extensive testing and analysis of 20,229 different observations on a wide variety of data ranging from population census figures to areas of rivers to baseball statistics. In 1938, Frank Benford published an article on his work in *The Proceeding of the American Philosophical Society*, and scientists began to pay more attention to his findings.

## Benford's Law Explained

Our number system uses the digits 0 through 9, and Benford was able to empirically demonstrate that in a large data set the probabilities for the distribution of digit frequencies would follow Table 1. By way of application, the first digit of a number is the leftmost digit and the first significant digit of zero is ignored. For example, in the number sequence 582, the number 5 is the first digit, 8 is the second digit, and 2 is the third digit.

A forensic accountant can employ this mathematical principle to execute several compelling analytical tests. Table 1 indicates that under Benford's Law, the expected proportion of numbers with a first digit being 1 is 0.30103 rounded to 30.1%, while the expected proportion for the numbers with the first digit being 2 is 0.17609 rounded to 17.6%, and so forth. As the numbers get higher in value, the percentage of their use becomes more infrequent. Therefore, the likelihood of obtaining any one of these nine numbers as the first significant digit is not evenly distributed, or with equal probability, at 11.1%, or with a 1 in 9 chance.

There was also a distribution of frequencies developed by Benford for the second, third, and fourth digits. Table 1 presents the expected digital frequencies for Benford's Law.

**Table 1**
**Benford's Law: Expected Digital Frequencies**

| | POSITION IN NUMBER | | | |
|---|---|---|---|---|
| Digit | 1st | 2nd | 3rd | 4th |
| 0 | | .11968 | .10178 | .10018 |
| 1 | .30103 | .11389 | .10138 | .10014 |
| 2 | .17609 | .10882 | .10097 | .10010 |
| 3 | .12494 | .10433 | .10057 | .10006 |
| 4 | .09691 | .10031 | .10018 | .10002 |
| 5 | .17918 | .09668 | .09979 | .09998 |
| 6 | .06695 | .09337 | .09940 | .09994 |
| 7 | .05799 | .09035 | .09902 | .09990 |
| 8 | .05115 | .08757 | .09864 | .09986 |
| 9 | .04576 | .08500 | .09827 | .09982 |

Source: Nigrini, M.J. 1996. A Taxpayer Compliance Application of Benford's Law: *The Journal of the American Taxation Association* 18:72-91.

A forensic accountant can utilize Benford's Law as a comprehensive tool to benchmark the results of the analysis of a target company's accounting records since the premise of this application is to analyze digit sequences in a set of numbers that follow a predictable pattern. If any numbers in a forensic accountant's testing of the target company's accounting records fall outside the Benford expected frequencies, more in-depth testing to the specific transactions can be implemented. A forensic accountant can be more efficient by examining a high level Benford's Law analysis, with the capability to drill down deeper into the original documents as required. By applying professional judgment and statistical techniques, a forensic accountant can identify anomalies that meet their criteria for further investigation.

Despite its origin many decades ago, Benford's Law was not recognized as a forensic accounting technique until the 1990s. Its current usage for assisting in the detection of fraud was formalized by Dr. Mark Nigrini, who is responsible for developing new applications to this old mathematical phenomenon for the field of forensic accounting in today's environment.

## Applying Benford's Law

The premise for Benford's application was that over time, individuals would invent numbers that tend to repeat their actions, and that people do not think like the natural digit patterns of Benford's Law. This conclusion is based upon the theory that human actions are not random, and that individuals will select numbers that they are accustomed to, or could easily create. Natural numbers, for this purpose, mean naturally occurring and not manipulated, and these types of numbers, generated in the ordinary course of business, have a tendency to follow Benford's Law. Therefore, the invented numbers by fraudsters, for non-existent goods or services for example, are unlikely to follow the sequence of Benford's Law. Alerted to this information, there are many practical applications for using Benford's Law in a financial investigation by a forensic accountant.

As the process counts digit sequences for values in the data set, the forensic accountant could apply analytical digital tests to a company's various accounting systems. As examples of the types of records that could be analyzed, the forensic accountant might select disbursement journals, invoices, cleared checks, deposits, transfers, or payment records. A forensic accountant would then compare the resulting target company output to the predicted results according to Benford's Law. For example, if artificial values are present in a data set, the distribution of the digits in this data set will likely exhibit a different shape, when viewed graphically, than the shape predicted by Benford's Law. The invented digit would appear more frequently than as expected by the Benford's Law results.

To better understand the subject, let's look at some of the digital analysis tests that can be deployed using Benford's Law during a forensic accountant's investigation. In applying these tests, the forensic accountant compares the output activity from selected accounting records of the target company to the expected digit frequency of Benford's Law. The digit tests consist of the following:

- *First-Digit Test* is a test for reasonableness that compares the actual first digit frequency distribution of a target company's data set to Benford's Law. At this test level, the data to be analyzed will be large and the test results would be used to identify obvious anomalies. This test will point a forensic accountant in the right direction, as fraudsters will tend to overuse certain digit patterns when inventing numbers.
- *Second-Digit Test* is similar in analysis to the First-Digit Test, and can assist a forensic accountant in identifying possible irregularities in the data set under investigation.
- *First-Two-Digit Test* is a more defined investigation than the two preceding tests, and uses the first two leading digits. For example, the first two digits of 7,380 are "73". There are 90 possible first two digit combinations ranging from the numbers 10 to 99. This test finds anomalies in the data that are not readily apparent from either the First-Digit or Second-Digit Tests as seen on their own.
- *First-Three-Digit Test* focuses on the 900 possible first three digit combinations of digits ranging from 100 to 999. This is a highly focused test similar to the First-Two-Digit Test, and should produce a smaller sample size to be tested.
- *Last-Two-Digit Test* is employed to find round numbers that may have been created to assist in a fraud. In the number sequence 7,500 the last two digits are "0" and "0," and may indicate rounded numbers that have been invented by the fraudster. This test is also useful in identifying patterns that may not be noticed when applying the previously mentioned tests.

In applying the tests outlined above, a forensic accountant would identify the target company's specific databases from an examination of its accounting records that required inspection. The practical application of each test must be examined with regard to the target company's overall volume of records, the specific assignment and the pertinent characteristics for which the forensic accountant intends to examine.

Suppose an analysis of a company's disbursement journals for a selected period was performed using the First-Digit Test. As long as the data set criteria applies to Benford's Law, the resultant list for first digit frequencies should match to Benford's Law as presented in Table 1. However, after performing a Benford's Law analysis of the disbursement journals' data set for the company, the number 6, as a first digit, appears 20% of the time. This would raise a red flag to a forensic accountant because Benford's Law predicts, as indicated in Table 1, that the number 6 would appear only 6.7% of the time. Therefore, this result points to the conclusion that there is a high probability that these numbers have been either invented or have some other abnormality in the data set.

Upon investigation, it might be detected that a fraudster negotiated unauthorized checks that start with the number 6 or, alternatively, a forensic accountant may discover a legitimate reason for the high occurrence of the number 6 as a first digit. It must be noted that not all anomalies are a result of fraudulent activities. Regardless of the results, the underlying investigation was performed on 100% of the data set with the forensic accountant's resources being allocated in a targeted and efficient manner. In many instances, just the follow up procedures of questioning company personnel about variations to the data set, and obtaining satisfactory clarification, can give a forensic accountant additional insight that would not have been obtainable by just looking at the basic number comparisons of totals and averages that were first presented.

Using the First-Digit Test, a forensic accountant may have identified many individual disbursements that need to be analyzed and, therefore, require the use of an alternate Benford Law digit test. Assume, for example, in the testing above there were 3,521 disbursements to be reviewed. Some examples of the range in numbers that required investigation employing the First-Digit Test included 6, 62, and 647.

In this example, to better focus the investigation, the forensic accountant would now perform the First-Three-Digit Test, and the only disbursement of the three examples listed above that necessitate an analysis would be the number 647. Therefore, after a further analysis using the First-Three-Digit Test by the forensic accountant, on the same data as the First-Digit Test, it is now determined that only 37 disbursements require additional investigation.

### Case Study

A company's senior management believes that there may be some fraudulent activities within the organization. Although sales have continued to increase each year, profits within certain divisions have not been following a similar trend. Management has retained a forensic accountant to perform a financial forensic investigation of its accounting records.

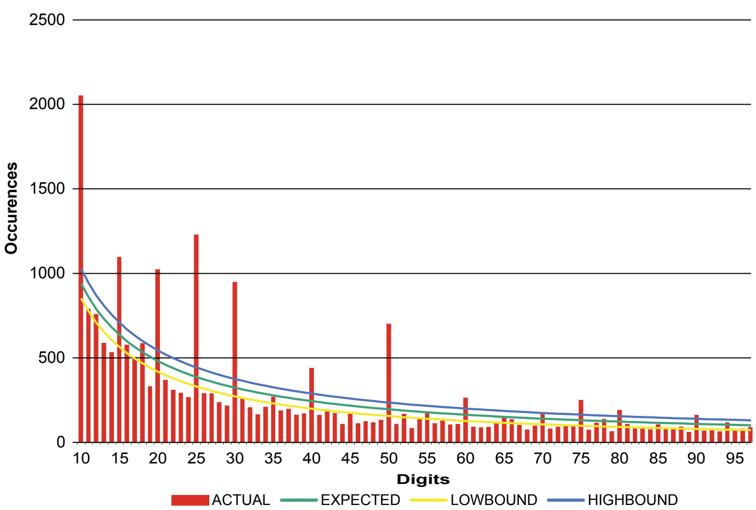
After examining the company's accounting systems, inspecting its available books and records, and interviewing key personnel, the forensic accountant determined that the company invoice amounts for a specific time period should be analyzed by comparing the company's results to Benford's Law using the First-Two-Digit Test. The output of this test, and its conclusions using the company's invoices data set, is then interpreted by the forensic accountant.

### Interpreting the Test Results

Table 2 presents a visual distribution of the results for the frequencies of all the digits analyzed by applying Benford's Law against 100% of the invoices for the inspection period of the target company. The results represent a comparison of the company's actual numerical distribution of invoice amounts with the expected numerical distribution applying Benford's Law.

The numbers across the bottom of Table 2 represent the number sequences that may exist for any of the company's invoice amounts in the period investigated. The left side of Table 2 lists the number of occurrences for each potential invoice amount with the two digit combination. The red bars in Table 2 represent the actual (observed) frequencies of the digits within the analyzed data set of the target company. The green line indicates the expected number of occurrences for a particular digit sequence according to Benford's Law. For additional information, the yellow and blue lines indicate the low and high range, respectively.

**Table 2**
**Benford's Law – Digital Frequencies**
**Invoice Amount Analysis**




### First-Two-Digits Test

For the target company under investigation, the forensic accountant can observe that the results from the First-Two-Digit Test show that the numbers 10, 15, 20, 25, 30, 40, and 50 are well above the distribution curve of Benford's Law expected frequencies. At this point, the forensic accountant has identified the areas where the Benford's Law distribution does not agree with the actual distribution of the company's invoices. Therefore, these invoices should be obtained and investigated further by analyzing each supporting document for these two digit combinations. The examination should uncover whether the variation is an acceptable abnormality or a problem invoice. If fraud is suspected, an investigation should follow to determine who is connected with the suspicious invoices.

It should be noted that in this investigation process, a forensic accountant must determine if any of the results are false-positives. Although a particular transaction was an anomaly, it was a proper company transaction not requiring further investigation as a suspicious item. To illustrate this point, a company that regularly disburses several monthly $2,512 lease payments had these items initially segregated for review by the forensic accountant as an anomaly, but during the analysis process these payments were identified as transactions occurring as part of the regular business operations of the company.

### Criteria for the Use of Benford's Law

Based upon the experience in testing that has been completed, certain criteria should be met for the data being tested to be a meaningful comparison to Benford's Law. The number patterns for the real-life data are natural, and independent from human or categorical control. The rules for data sets should meet these conditions, as not all data would conform to Benford's digit frequency. Therefore, the data set examined should:

- Describe the sizes of similar data, such examples include the lengths of rivers and populations of cities;
- Have no artificial limits, as well as no minimum or maximum amounts, such as the height of adults or a minimum bank charge;
- Not have numbers that are pre-arranged or assigned, such as Social Security numbers;
- Have more smaller items than larger items, such as there are more smaller cities than larger cities;
- Not be a subset of another set;
- Not represent aggregated totals, such as monthly invoices;
- Have the accounting data relate to a specific accounting period, such as one year;
- Be large enough so that the expected proportions can manifest themselves.

### Conclusion

With the continued growth and complexity of companies, databases, and accounting systems, fraudulent activities are becoming more difficult to detect without the help of sophisticated analytical tools. Benford's Law is a powerful tool for assisting the forensic accountant in uncovering fraudulent activities within a company's accounting records. The forensic accountant can deploy the analytical tests discussed in this article to measure the expected population distribution of digits against the actual distribution. These tests allow a forensic accountant to isolate and focus on target groups of data. Therefore, a forensic accountant can be more productive by concentrating on identifying errors or unusual transactions that have a high probability of being either fraud, errors, inefficiencies, or other unexplained variations within the target company.

Deviations from Benford's Law might signal accounting errors, invented numbers, biases, or other irregularities in the data set. The use of digital analysis software enables the forensic accountant to efficiently import data, join various company databases, analyze large amounts of financial information across different company files, extract data with specific characteristics for further analysis, and print reports that contain data analysis results. Ultimately, using various Benford's Law techniques, professional judgment, and investigative skills, forensic accountants can work smarter and more efficiently through the examination process.

*Mark S. Warshavsky, CPA/ABV/CFF, CVA, CBA, CFE, DABFA, CFFA, is Partner-in-Charge of the Business Valuation and Litigation Support Group at the New York firm of Gettry Marcus Stern & Lehrer, CPA, P.C. He has been retained as a consultant and expert in the areas of litigation support, forensic accounting, and business valuations for both Federal and state cases. He also serves on the editorial advisory board of several national publications for which he peer reviews articles submitted for publication in the areas of business valuation, forensic accounting, and litigation support. He is a founding member, author, and instructor for NACVA's Forensic Accounting Academy. Mr. Warshavsky can be reached at mwarshavsky@gmslnv.com or 516-364-3390 ext. 121.*